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A Scan-Based Hierarchical Heuristic Optimization Algorithm for PCB Assembly Process

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***Abstract*—Surface mount technology is essential to the development of the electronic manufacturing industry. This article studies optimizing the surface mount process for the beam-head placement machine. A mixed-integer programming (MIP) model is proposed for this problem, which is decomposed into three interconnected hierarchical parts: feeder allocation; component assignment; and pick-and-place (PAP) sequence problems. This article proposes an efficient hierarchical framework with three elaborately designed heuristics to solve the above problem. The design of the scan-based algorithms optimizes the subobjectives of feeder allocation and component assignment. First, the allocation heuristic arranges the feeders into slots as a prerequisite for other problems. Then, the component assignment heuristic determines the component type for each head with a variety of criteria and long short-term objectives. Finally, the PAP sequence problem is solved using a modified beam search algorithm. The proposed algorithm offers advantages in terms of effectiveness, efficiency, and extension, which can satisfy various customization demands. Experiments are conducted on our self-designed placement machine using industrial and randomly generated data. Computational experiments show that the scan-based heuristic algorithm obtains near-optimal solutions with a gap of 9.93% averagely compared with the proposed MIP model and provides efficiency improvement over the mainstream studies.**

***Index Terms*—Hierarchical decomposition, mixed-integer linear model, printed circuit board (PCB) assembly optimization, scan-based heuristic.**

I. Introduction

NOWADAYS, the widespread use of electronic products in modern life has raised attention to the price and quality of printed circuit boards (PCBs). A complicated collection of production procedures makes up for manufacturing electronic products. PCB assembly is one of the necessary but time-consuming processes among them. The placement machine is a sophisticated computer-controlled apparatus that integrates mechanical, electrical, and optical techniques [1]. The factory uses automatic manufacturing lines to produce high-quality PCB, and the maximum production capacity of the placement machine is the efficiency bottleneck of the whole production line. The application benefit of an effective assembly optimization technique is enormous.

This article focuses on the beam-head placement machine, which has a stationary PCB platform, two stationary feeder bases, and a moving gantry with beam heads, as shown in Fig. 1. The feeders loading with components are installed on the feeder base. There are three basic types of feeders for assembling various package component parts: tape, stick, and tray. The gantry moves between the PCB and the feeder base to pick and place the components with vacuum valves. The fly camera is equipped in the heads for chip detection; for some large chips, the gantry moves to the fixed camera for inspection. An auto nozzle changer (ANC) is kept with multiple nozzle types to fulfill the assembly needs for various component shapes. The primary distinction between the beam-head placement machine and other types is its mechanical design, which enables multiheads to pick up components from feeders simultaneously.

As shown in Fig. 2, the surface mount process consists of six different types of operations, and the dashed line framed part includes a pick-and-place (PAP) cycle, which is the fundamental unit. The nozzle change, component pickup, and component placement operations in a PAP cycle take substantial time, and

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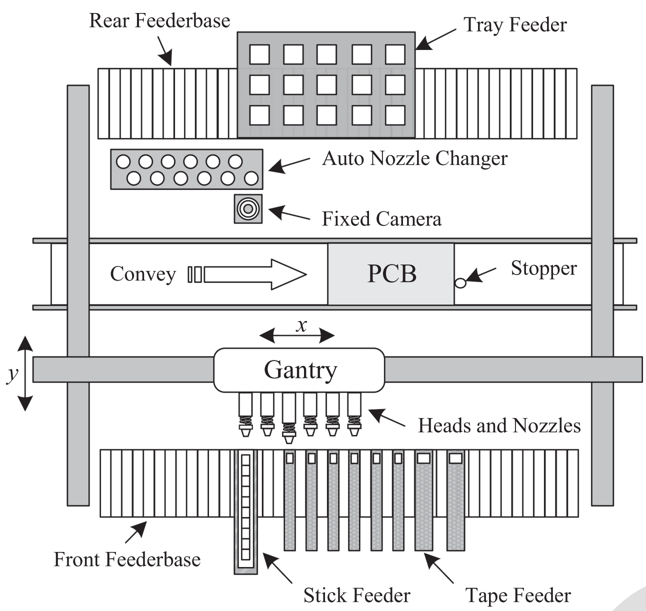

Fig. 1. Layout of the beam-head placement machine.

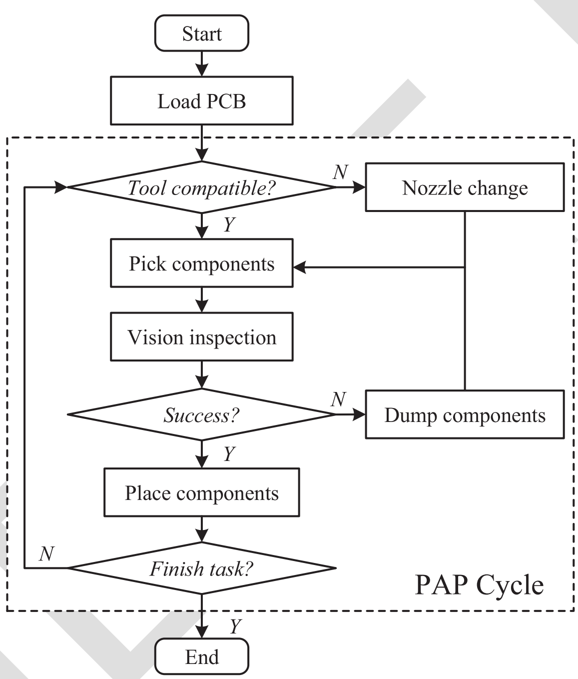

Fig. 2. Workflow chart of the surface mount process.

the algorithm can optimize the first two operations. More specifically, by combining multiple head motions, the pickup operation could be more effective, and the nozzle changes are connected to the sequence of component pickup. The component dumping operations caused by image processing errors are exceptions and are not considered in this article.

The surface mount optimization problem has multiple variables with significant coupling and is an NP-hard problem [2]. A general technique for this complex optimization problem is the hierarchical decomposition method [3], [4]. This challenging combinatorial optimization problem is made up of the location problem, the assignment problem, and the route schedule problem. In the locating problem, the depots are feeders assigned for the assembling process [5]. The assignment problems are concerned with determining the type of component picked up by the placement head, which must take into account the tool compatibility of the nozzle component [6], [7] and its influence on simultaneous pickup [8], [9], both of which are essential factors influencing assembly efficiency. The PAP route schedule is covered in studies [10] and [11] utilizing heuristic and mathematical programming, respectively.

Surface mount optimization has been solved by a variety of algorithms, such as mathematical programming, evolutionary algorithms, tailored constructive heuristics, etc. The mathematical programming method is limited by the complexity of the problem, and the subproblem is the subject of multiple studies [12], [13], [14]. Medium-size problems can be solved using mathematical programming combined with the aggregation [6] approach and the augmented technique for multiobjective [15]. Evolutionary algorithms have been widely used in surface mount optimization problems [16], [17], [18], [19], [20], such as the genetic algorithm, particle swarm algorithm, shuffle leapfrog method, etc. Complex optimization problems may have multiple subobjectives, and research has been done to combine multiobjective optimization with evolutionary algorithms to find the Pareto fronts of the problems [21], [22]. Some studies provide constructive heuristics, which solve the problem based on the structure of the problem and significantly improve the quality of solutions [23].

To summarize, there is still a long way to industrial deployments. Current research is flawed by irrational assumptions or inadequate examination of the factors influencing assembly efficiency. The algorithm needs to be constructed to work in various application scenarios. In this article, we propose a mathematical model and a novel hierarchical scan-based heuristic framework for the surface mount optimization problem. The contributions of this article are summarized as follows.

1) A mixed-integer model for the PCB assembly process is proposed. The model fully incorporates the factors affecting assembly efficiency and decomposes the assembly process into pickup and placement parts. The pickup model takes into account the impact of simultaneous pickup on efficiency for the first time, and the placement model is modeled as a variant of the multiple traveling salesman problem (MTSP).
2) The hierarchical decomposition approach reduces the complexity of the problem. Based on the problem characteristics for each subproblem, three elaborately designed heuristics combined with the scanning concept are proposed, which can obtain a nearly optimal solution and perform better on the search efficiency compared to other approaches.
3) The proposed algorithms demonstrate substantial extensions, which are adaptable enough to satisfy the operators' various customized requirements. The algorithm optimization process simulates the pickup process, which can

be adapted to the actual situation of the feeder allocation and component pickup operation tasks.

The rest of this article is organized as follows. Section II presents a mixed-integer mathematical model, and Section III proposes a scan-based hierarchical heuristic algorithm to provide a satisfying PCB assembly solution. In Section IV, the experiment results are introduced and compared with the mainstreaming study. Finally, Section V concludes this article.

II. Mathematical Model

A. Problem Description

The surface mount optimization is to solve the scheduling problem of the PCB assembly process and get an efficient solution with complicated constraints and multiple decision variables. The typical subproblems of the assembly process are the feeder allocation problem and the head task assignment problem. The former solves the problem of the arranged slots of feeders, while the latter determines the assembly sequence. The component assignment problem and the PAP route schedule problem are further decompositions in this article for the head task assignment. There is a progressive relationship between the two subproblems, and the complexity of the problem can be reduced by determining the component type and then the placement point of each head.

The underlying subproblems are tightly coupled. The feeder allocation affects component assignment for maximizing the number of simultaneous pickups, i.e., combining more pickup operations. The pickup slots of the assembly process and the assembly sequence determine the overall movement distance of the gantry. There may be redundant movements for pickup operations and nozzle changes for the consistency of the nozzle type, component type, and feeder slot.

This article makes the following assumptions about the optimization problem with litter impact on the optimality of the solution.

1) The X- and Y-axis motor movement is simplified to an independently controlled trapezoidal profile.
2) The interval distance between adjacent heads is integer times the interval distance between two adjacent feeder slots.
3) Only an appropriate type of nozzle can pick up the component.
4) The ANC configuration is predetermined, and the movement at different holes is ignored.
5) Tray and stick feeders have predetermined arrangements and are not incorporated into the optimization process.

B. Optimization Objective and Constraints

The surface mount process is accomplished by a complex series of motions that work together. The target of minimizing the assembly time depends on the distance of the gantry traveling, the number of pickup operations, and the number of nozzle change operations, which are the subobjectives. The coupling of subobjectives is reflected in combining the pickups of multiple heads, which may bring additional nozzle change, and the distance of the gantry traveling relies on the pickup and nozzle change operations.

The constraints for surface mount optimization problems can be divided into four categories: job completion constraint, mechanical restriction, tool requirement, and artificial constraints. Job completion is essential for surface mount tasks, and each component must be assembled accurately on the corresponding PCB pads. The mechanical restriction concerns the structural characteristics of the placement machine, such as each head having unreachable pickup slots. Another type of mechanical constraint is positional interference caused by feeders occupying multiple slots. The restricted number of nozzles and feeders available will also impede optimizing assembly efficiency. Tool consistency is a critical assurance for the assembly process. In terms of artificial limits, operators may want to prearrange feeders, prohibit some feeder slots, and set prohibited heads.

C. Mixed-Integer Programming (MIP) Model

Mathematical programming methods to solve the surface placement task must deal with the problems of numerous decision variables and intricate constraints. The route scheduling of the gantry is constrained by the type of component, nozzle, and slot that corresponds to each head, which greatly increases the complexity of the model. This article proposes a hierarchical MIP model to solve the problem effectively by decomposing the surface mount process into two parts: the pickup model and the placement model. The pickup model is a prerequisite for the solution of the placement model, which determines the movement time parameters and the placement head task in the placement model. The notations of the proposed model are shown in Table I. The table describes the type of decision variables, all of which are nonnegative.

1) Pickup Model:

$$\min t_c \cdot \sum_{k\in K} g_k + t_n \cdot \sum_{h\in H} d_h + t_p \cdot \sum_{s\in S'} \sum_{k\in K} e_{sk} + t_m \cdot \sum_{k\in K} u_k \tag{1}$$

$$g_k \geq g_{k+1} \quad \forall k \in K \backslash \{|K|\} \tag{2}$$

$$\sum_{i\in I} \sum_{s\in S} x_{iskh} \leq g_k \quad \forall k \in K, h \in H \tag{3}$$

$$\sum_{j\in J} \sum_{i\in I} \sum_{s\in S} \mu_{ij} \cdot x_{iskh} \leq 1 \quad \forall k \in K, h \in H \tag{4}$$

$$\sum_{s\in S} \sum_{h\in H} \sum_{k\in K} x_{iskh} = \psi_i \quad \forall i \in I \tag{5}$$

$$d_h = \frac{1}{2} \sum_{k\in K\backslash\{|K|\}} \left(\sum_{j\in J} \left| \sum_{i\in I} \sum_{s\in S} \mu_{ij} \cdot x_{iskh} - \sum_{i\in I} \sum_{s\in S} \mu_{ij} \cdot x_{is(k+1)h} \right| - 1 \right) \quad \forall h \in H \tag{6}$$

$$e_{sk} \leq \sum_{i\in I} \sum_{h\in H_s} x_{i[s+(h-1)\cdot\tau]kh} \leq M \cdot e_{sk} \quad \forall s \in S', k \in K \tag{7}$$

TABLE I
SUMMARY OF NOTATIONS

	Notation	Description
Indexes & Sets	$i \in I$	Index of component type, $I = \{1, 2, \cdots\}$
	$j \in J$	Index of nozzle type, $J = \{1, 2, \cdots\}$
	$p, q \in P$	Index of (placement) point, $P = \{1, 2, \cdots\}$
	$h, l \in H$	Index of head, $H = \{1, 2, \cdots\}^1$
	$a \in A$	Index of arc, $A = \{(h, l) \mid h \neq l, h, l \in H\}^2$
	$k \in K, K'$	Index of cycle, $K = \{1, 2, \cdots\}$, $K' = \{k \mid g_k > 0\}$
	$s, r \in S$	Index of slot, $S = \{1, 2, \cdots\}^3$
Parameters	t_c	Average moving time of round trip between PCB and feeder base
	t_n	Average time of nozzle change operation
	t_p	Average time of pickup operation
	t_m	Average moving time on the feeder base per slot
	μ_{ij}	Compatibility of component type i and nozzle type j
	η_{ip}	Correspondence of component type i and point p
	ψ_i	Number of points of component type i
	ϕ_i	Feeder number of component type i available
	ζ_j	Number of nozzle type j available
	λ^{FW}_{pkh}	Moving time from feeder base to the first point in cycle k
	λ^{PL}_{pqa}	Moving time between point p and point q along with arc a
	λ^{BW}_{pkh}	Moving time from the last point to feeder base in cycle k
	ρ	Interval distance between adjacent heads
	τ	Interval ratio of adjacent heads and adjacent slots
	M	Sufficiently large number
Decision Variables[4]	g_k	Binary variable $= 1$ iff at least one point is picked up and placed in cycle k
	u_k	Integer variable the pick-up moving slot of cycle k
	d_h	Integer variable the number of nozzle changes of head h
	e_{sk}	Binary variable $= 1$ iff component is picked up from the equality slot s in cycle k
	f_{si}	Binary variable $= 1$ iff comp. type i is assigned to slot s
	x_{iskh}	Binary variable $= 1$ iff head h picks up component type i from slot s in cycle k
	w_{pqka}	Binary variable $= 1$ iff point p is placed after point q along with arc a in cycle k
	y_{pkh}	Binary variable $= 1$ iff point p is the first point placed with head h in cycle k
	z_{pkh}	Binary variable $= 1$ iff point p is the last point placed with head h in cycle k

[1] The head set H_s is the subset H, which contains the heads that can pick up components from slot s, $H_s = \{\max(1, -\lfloor (s-1)/\tau \rfloor + 1), \cdots, \min(|H|, \lceil (|S| - s + 1)/\tau \rceil) + 1\}$

[2] The arcs of A represent the placement sequence of the heads. A_h, A_h^f and A_h^t are subsets of A, where A_h has the arcs of A that pass head h, A_h^f has the arcs of A from head h, and A_h^t has the arcs of A to head h.

[3] S' is the set of equality slot index, which refers to the left-most head aligned slot, $S' = \{-\tau \cdot (|H| - 1) + 1, \cdots, 1, 2, \cdots, |S|\}$

[4] The intermediate continuous variables v_{pq}, n_p, and m_p are used to eliminate subtour.

$$u_k \geq s \cdot e_{sk} - r \cdot e_{rk} \quad \forall k \in K, s, r \in S' \tag{8}$$

$$f_{si} \leq \sum_{h \in H} \sum_{k \in K} x_{iskh} \leq M \cdot f_{si} \quad \forall s \in S, i \in I \tag{9}$$

$$\sum_{i \in I} f_{si} \leq 1 \quad \forall s \in S \tag{10}$$

$$\sum_{h \in H} \sum_{i \in I} \sum_{s \in S} \mu_{ij} \cdot x_{iskh} \leq \zeta_j \quad \forall k \in K, j \in J \tag{11}$$

$$\sum_{s \in S} f_{si} \leq \phi_i \quad \forall i \in I. \tag{12}$$

The objective of the pickup model (1) consists of four terms: the number of cycles; nozzle change operations; pickup operations; and pickup moving distance; where the pickup moving distance is represented by the number of slots the gantry crosses over. Constraint (2) ensures that the first few cycles of the surface mount process are given top priory for completion. The heads and work cycle are consistent with Constraint (3). Constraint (4) ensures that each head is equipped with at most one nozzle type. The completion of the surface mount process for each component type is guaranteed by constraint (5). Constraint (6) calculates the number of nozzle changes of each head, and constraint (7) converts the pickup slot of each head to the leftmost head to calculate the number of the pickup operations in each cycle. Constraint (8) calculates the number of slots crossed over by the gantry for the pickup process in each cycle. Constraint (9) ensures the consistency of head pickup operations and feeder allocation. Constraint (10) ensures that each slot is assigned at most one feeder. Constraints (11) and (12) indicate the limited number of available nozzles and feeder base, respectively.

2) Placement Model:

$$\min \sum_{k \in K'} \left\{ \sum_{p \in P} \sum_{h \in H} \lambda^{FW}_{pkh} \cdot y_{pkh} + \sum_{p \in P} \sum_{q \in P} \sum_{a \in A} \lambda^{PL}_{pqa} \cdot w_{pqka} + \sum_{p \in P} \sum_{h \in H} \lambda^{BW}_{pkh} \cdot z_{pkh} \right\} \tag{13}$$

$$\sum_{q \in P} \sum_{a \in A_h} w_{pqka} = \sum_{i \in I} \sum_{s \in S} \eta_{ip} \cdot x_{iskh} \quad \forall p \in P, k \in K', h \in H \tag{14}$$

$$\sum_{p \in P} \sum_{q \in P} \sum_{a \in A_h} w_{pqka} \leq 2 \quad \forall k \in K', h \in H \tag{15}$$

$$\sum_{p \in P} (y_{pkh} + z_{pkh}) \leq 1 \quad \forall k \in K', h \in H \tag{16}$$

$$\sum_{q \in P} \sum_{a \in A_h^t} w_{qpka} + y_{pkh} = \sum_{q \in P} \sum_{a \in A_h^f} w_{pqka} + z_{pkh} \quad \forall k \in K', h \in H, p \in P \tag{17}$$

$$y_{pkh} \leq \sum_{q \in P} \sum_{a \in A_h^f} w_{pqka} \quad \forall k \in K', h \in H, p \in P \tag{18}$$

$$z_{pkh} \leq \sum_{q \in P} \sum_{a \in A_h^t} w_{qpka} \quad \forall k \in K', h \in H, p \in P \tag{19}$$

$$\sum_{p \in P} \sum_{h \in H} y_{pkh} = 1 \quad \forall k \in K' \tag{20}$$

$$\sum_{p \in P} \sum_{h \in H} z_{pkh} = 1 \quad \forall k \in K' \tag{21}$$

$$\sum_{k \in K'} \left(\sum_{h \in H} y_{pkh} + \sum_{q \in P} \sum_{a \in A} w_{pqka} \right) = 1 \quad \forall p \in P \tag{22}$$

$$\sum_{k\in K'}\left(\sum_{h\in H} z_{pkh}+\sum_{q\in P}\sum_{a\in A} w_{qpka}\right)=1 \quad \forall p\in P \tag{23}$$

$$m_p+\sum_{q\in P} v_{pq}-n_p-\sum_{q\in P} v_{qp}=1 \quad \forall p\in P \tag{24}$$

$$v_{pq}\le \sum_{k\in K'}\sum_{a\in A}(|P|-|K'|+1)\cdot w_{pqka} \quad \forall p,q\in P \tag{25}$$

$$n_p\le \sum_{k\in K'}\sum_{h\in H}(|P|-|K'|+1)\cdot y_{pkh} \quad \forall p\in P \tag{26}$$

$$m_p\le \sum_{k\in K'}\sum_{h\in H}(|P|-|K'|+1)\cdot z_{pkh} \quad \forall p\in P. \tag{27}$$

The objective of placement model (13) is the total of the moving times except for the pickup movement, which has been solved in the pickup model. The parameters of moving time λ in the objective are obtained based on the solution of the pickup model. Constraint (14) ensures that the solutions of the pickup model and the placement model are consistent. Constraints (15) and (16) ensure that each head is placed at most one placement point. Constraints (17)–(19) ensure the continuity of the placement task, i.e., the placement head is unique for each point. Constraints (20) and (21) mean that the path of the placement head from the feeder base to the PCB and from the PCB back to the feeder base is unique for each cycle. Constraints (22) and (23) ensure that the entry edge and the leave edge of each point are unique, respectively. Constraints (24)–(27) are utilized to eliminate the subtour for each cycle.

The pickup model (1)–(12) and placement model (13)–(27) involve an assignment problem and a restricted MTSP problem, which are two well-known NP-hard problems. Therefore, the proposed model above can be solved only for small-scale data in a reasonable amount of time. In Section III, we will further decompose the problem following the optimization objective, and an efficient hierarchical framework will be proposed to solve this problem.

III. Hierarchical Heuristic Optimization

A. Scan-Based Heuristic Hierarchical Framework

Hierarchical decomposition is a common method for solving complicated optimization problems. A direct solution to the whole problem may bring on a dimensionality disaster because of the numerous constraints and decision variables. It makes sense to design the algorithm by the relevance of the subobjective. The constructive scan heuristic algorithm [5] is the basis of the proposed method in this article, which overcomes the shortcomings of the lengthy solving time and greedily maximizes the pickup efficiency.

This article decomposes the surface mount optimization problem into the feeder allocation problem, component assignment problem, and PAP sequence problem. We prioritize feeders since ignoring them will significantly increase pickup operations and longer moving routes. Furthermore, if the feeder arrangement must be changed each time the PCB changes, the labor cost associated with reoptimizing the algorithm could increase. The PAP route schedule is the final subproblem to be solved since the moving distance of the placement heads has less impact on assembly efficiency than other factors.

The relationship among subproblems, subobjectives, and constraints is shown in Fig. 3. The feeder allocation and component assignment problems impact the nozzle changes and simultaneous pickups, while the route schedule problem is relatively independent. It can be expected that there are certain similarities in the algorithm design of feeder allocation and component assignment. The superscripts NZ, CP, and PT of the notations in the algorithm description are the abbreviation of nozzle type, component type, and the number of placement points, respectively.

Algorithm 1: Feeder Allocation Heuristic.

Input: PCB data, component data, feeder data, and nozzle pattern $\mathcal{N}$
Output: feeder assignment $\mathcal{F}^{\mathrm{CP}}$ and $\mathcal{F}^{\mathrm{PT}}$
Initialize $\mathcal{F}^{\mathrm{CP}}$ as the component type prearranged on the feeder base (-1 for empty), $\mathcal{F}^{\mathrm{PT}}$ as the number of the placement points, and $\mathcal{S}$ as an empty stack;
while $\sum_{i\in I}\psi_i\neq 0$ **do**
  Initialize $V_b\leftarrow 0$ as the best allocation value;
  for $s\leftarrow 1$ to $|S|-(|H|-1)\cdot\tau$ **do**
    foreach $s'=s+(h-1)\cdot\tau, h\in H$ **do** $\mathcal{H}^{\mathrm{CP}}(h)\leftarrow\mathcal{F}^{\mathrm{CP}}(s'), \mathcal{H}^{\mathrm{PT}}(h)\leftarrow\mathcal{F}^{\mathrm{PT}}(s')$ $I'\leftarrow I$;
    for $j\leftarrow\mathcal{N}(h), h\in\{h'|\mathcal{H}^{\mathrm{CP}}(h')>0\}$ **do**
      if $\psi_i=0, \forall i\in\{i'|\xi_{i'j}\neq 0, i'\in I'\}$ **then**
        push $i\leftarrow\mathrm{argmax}_{i'\in I'}\{\psi_{i'}\}$ into $\mathcal{S}$;
      else
        $i\leftarrow\mathrm{argmax}_{i'\in I'}\{\psi_{i'}|j\cdot\xi_{i'j}>0\}$ $\mathcal{H}^{\mathrm{CP}}(h)\leftarrow i, \mathcal{H}^{\mathrm{PT}}(h)\leftarrow\psi_i$;
      end
      $I'\leftarrow I'\backslash\{i\}$;
    end
    Pop components from $\mathcal{S}$ and assign them to the heads $h\in\{h'|\mathcal{H}^{\mathrm{PT}}(h')=0\}$;
    if $\sum_{h\in H}\mathcal{H}^{\mathrm{PT}}(h)>V_b$ **then**
      $V_b\leftarrow\sum_{h\in H}\mathcal{H}^{\mathrm{PT}}, \mathcal{H}_b^{\mathrm{PT}}\leftarrow\mathcal{H}^{\mathrm{PT}}, \mathcal{H}_b^{\mathrm{CP}}\leftarrow\mathcal{H}^{\mathrm{CP}}$, $s_b\leftarrow s$
    end
  end
  $\delta=\min\{\mathcal{H}_b^{\mathrm{PT}}(h)|\mathcal{H}_b^{\mathrm{PT}}(h)\neq 0, h\in H\}$;
  for $s'\leftarrow s_b+(h-1)\cdot\tau, h\in H$ **do**
    if $\mathcal{F}^{\mathrm{PT}}(s')=-1$ **then**
      $\mathcal{F}^{\mathrm{CP}}(s')\leftarrow\mathcal{H}_b^{\mathrm{CP}}(h)$;
    end
    $\mathcal{F}^{\mathrm{PT}}(s')\leftarrow\mathcal{F}_b^{\mathrm{PT}}(s')-\delta$;
    $\mathcal{N}(h)\leftarrow j, \psi_i\leftarrow\psi_i-\delta$ where $i=\mathcal{H}_b^{\mathrm{CP}}(h)$, $j=\sum_{j'\in J}j'\cdot\xi_{ij'}$;
  end
end

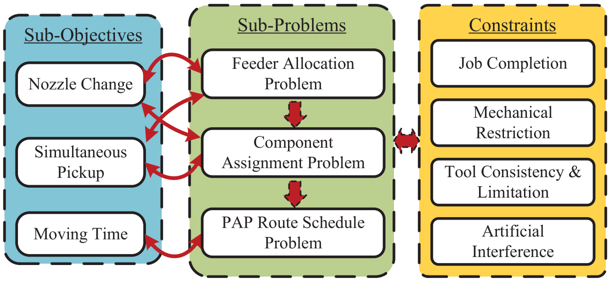

Fig. 3. Relationship of surface mount process optimization subobjectives, subproblems, and constraints.

B. Feeder Allocation Heuristic Algorithm

Feeder allocation is a prerequisite for other subproblems, and an appropriate arrangement will significantly enhance pickup efficiency, which determines the component pickup slot. The basic idea of feeder allocation heuristic described in Algorithm 1 is assigning the feeders while scanning the feeder base under the constraint of the nozzle pattern, which can maximize the number of pickup points allocated in a round and avoid nozzle change. The algorithm assigns feeders to the empty slots in the different rounds, reserving the component types already arranged in the head-aligned slots. The component types that can be allocated in the head-aligned slots are determined by the nozzle pattern. The nozzle pattern helps to reduce the number of nozzle changes for subsequent pickup operations. The type of component with more placement points that do not meet the nozzle pattern restriction is stored in component stacks to guarantee a comparably concentrated position of the feeder allocation. At the end of the assignment, the algorithm assigns components in the stack to slots.

C. Component Assignment Heuristic Algorithm

The algorithmic framework for feeder allocation and component assignment is similar, and both are based on heuristic scanning. The feeder allocation solves the problem of component pickup position, and the component assignment solves the problem of pickup sequence. The scanning heuristic efficiently optimizes the simultaneous pickups, which significantly reduces the overall pickup operations by integrating the pickup operations of multiheads. Similar to feeder allocation produces, each head aligns to a slot starting from different pickup slots, the component assigned to the head should satisfy the following criteria.

1) *Pickup feasibility:* The head-aligned slot contains unpicked placement points.
2) *Pickup constraint:* The head-equipped number of nozzles does not exceed the number available.
3) *Pickup prejudgment:* The component being picked up does not lessen the number of subsequent simultaneous pickups of the prejudgment.
4) *Pickup objective:* The efficiency gain from pickup outweighs the efficiency loss from nozzle change.

Algorithm 2: Component Assignment Heuristic.

Input: PCB data, feeder allocation $\mathcal{F}^{\mathrm{CP}}$ and $\mathcal{F}^{\mathrm{PT}}$
Output: component assignment $\mathcal{C}$ and cycle group $\mathcal{K}$
Initialize a $1 \times |H|$ matrix $\mathcal{M}$ of *None* as the initial nozzle assignment;
while $\sum_{s \in S} \mathcal{F}^{\mathrm{PT}}(s) \neq 0$ **do**
  Initialize $V_b \leftarrow 0$ as the best assignment value;
  for $\mathcal{N} \in \mathcal{M}$, $s \leftarrow 1$ to $|S| - (|H|-1)\tau$ **do**
    for $h \in H$ **do**
      $s' \leftarrow s + (h-1)\tau$, $i \leftarrow \mathcal{F}^{\mathrm{CP}}(s')$;
      Calculate $v \leftarrow e_1 \cdot v_1 - e_2 \cdot v_2$ where
      $v_1 = \min_{h' \in H}\{\mathcal{H}^{\mathrm{PT}}(h') > 0\} \cup \{\mathcal{F}^{\mathrm{PT}}(s')\}$,
      $v_2 = \sum_{h' \in H} |\mathcal{N}(h') - \sum_j \xi_{\mathcal{H}^{\mathrm{PT}}(h') \cdot j}|$;
      if $\mathcal{F}^{\mathrm{PT}}(s') > 0$ **and** $v > 0$ **then**
        $\mathcal{H}^{\mathrm{CP}}(h) \leftarrow \mathcal{F}^{\mathrm{CP}}(s')$, $\mathcal{H}^{\mathrm{PT}}(h) \leftarrow \mathcal{F}^{\mathrm{PT}}(s')$;
      end
    end
    Calculate short-term objective V_s and long-term objective V_l with Algorithm 3;
    if $e \cdot V_l + (1-e) \cdot V_s > V_b$ **then**
      $V_b \leftarrow e \cdot V_l + (1-e) \cdot V_s$, $s_b \leftarrow s$;
      $(\mathcal{H}_b^{\mathrm{PT}}, \mathcal{H}_b^{\mathrm{CP}}, \mathcal{H}_b^{\mathrm{NZ}}) \leftarrow (\mathcal{H}^{\mathrm{PT}}, \mathcal{H}^{\mathrm{CP}}, \mathcal{H}^{\mathrm{NZ}})$
    end
  end
  $k \leftarrow \min_{h \in H}\{\mathcal{H}_b^{\mathrm{PT}}(h) > 0\}$;
  foreach $h \in H$ **do**
    $s' \leftarrow s_b + (h-1) \cdot \tau$, $\mathcal{F}^{\mathrm{PT}}(s') \leftarrow \mathcal{F}^{\mathrm{PT}}(s') - k$
  if $\mathcal{H}_b^{\mathrm{PT}}(h) > 0$ **or** $\mathcal{F}^{\mathrm{PT}}(s) = 0, \forall h \in H, s \in S$ **then**
    Attach $\mathcal{H}_b^{\mathrm{CP}}$ to $\mathcal{C}$, $\mathcal{H}_b^{\mathrm{NZ}}$ to $\mathcal{M}$, k to $\mathcal{K}$ along with column direction;
  end
end

Algorithm 2 describes the implementation of the component assignment. Each round determines the type of component assigned to heads with unpicked placement points and the related cycle groups. A "cycle group" is a set of consecutive PAP cycles with the same component assignments. It should be mentioned that the scanning-based pickup procedure tries to maximize the number of simultaneous pickups while minimizing the expense of nozzle changes. The component assignment heuristic is forward looking, which means that the single-head component assignment prejudges its impact on subsequent assignments. This is principally reflected in the following two aspects: the first is to assign just those components that improve the overall objective, and the second is the long short-term objectives. As for long short-term objectives implemented in Algorithm 3, the long-term objective is to simultaneously pick up components from all the aligned slots until one is empty, while the short-term goal is to pick up all the components from the aligned slots greedily. The current component assignment result is the short-term objective, and its effect on pickup efficiency as a whole is the long-term objective. The long short-term objective is the weighted sum of these two.

Algorithm 3: Long Short-Term Objective Calculation.

Input: Head component assignment $\mathcal{H}^{\text{PT}}$
Output: short-term objective V_s and long-term objective V_l

Initialize short-term objective $V_s \leftarrow 0$ and long-term objective $V_l \leftarrow -e_2 \cdot \sigma$;
$V_s \leftarrow e_1 \cdot \omega \cdot \min_{h' \in H}\{\mathcal{H}^{\text{PT}}(h') > 0\} - e_2 \cdot \sigma$ where $\omega = |H| - |\{h' | \mathcal{H}^{\text{PT}}(h') > 0, h' \in H\}| - 1$ and $\sigma = \sum_{h' \in H} |\mathcal{N}(h') - \sum_{j \in J} j \cdot \xi_{\mathcal{H}^{\text{CP}}(h') \cdot j}|$;
while $\mathcal{H}^{\text{PT}}(h) > 0, \exists h \in H$ **do**
  $V_l \leftarrow V_l + e_1 \cdot \omega \cdot \min_{h' \in H}\{\mathcal{H}^{\text{PT}}(h') > 0\}$ where $\omega \leftarrow |H| - |\{h' | \mathcal{H}^{\text{PT}}(h') > 0, h' \in H\}| - 1$;
  $\mathcal{H}^{\text{PT}} \leftarrow \mathcal{H}^{\text{PT}} - \min_{h' \in H}\{\mathcal{H}^{\text{PT}}(h') > 0\}$;
  foreach $h' \in H$ **do** $\mathcal{H}^{\text{NZ}}(h') \leftarrow \sum_{j \in J} j \cdot \xi_{\mathcal{H}^{\text{CP}}(h') \cdot j}$
end

D. PAP Sequence Heuristic Algorithm

The pick and placement route schedules make up the PAP route schedule problem. In case the feeder allocation and the component assignment are determined, the pickup procedure calls for picking up components from each preset slot in a single direction on the feeder base. Algorithm 4 shows the process of beam search, which is utilized to solve the placement route schedule problem by retaining multiple potentially optimal solutions based on greedy search. The placement process can be thought of as a constrained vehicle route schedule problem with capacity constraints and candidate placement point constraints imposed by the component assignment. The dynamic programming is employed to determine the placement sequence in each cycle, which is efficient with a limited number of placement points.

E. Extension of the Proposed Algorithm

The proposed algorithms show significant applicability expansion. First, the algorithm may balance the nozzle change and pickup operation cost by modifying the parameter weights. Second, regardless of the number of linear-aligned heads, the technique may be utilized to achieve simultaneous pickup. Even though the adjacent interval distance ratio between heads and slots is not always an integer, the approximate value also improves productivity by shortening the pickup distance of the gantry. Finally, since the algorithm implementation is essentially a simulation of the picking process, it can be fine-tuned to offer a tailored solution, including but not limited to preassign feeders, assigning nozzle to head, and prohibiting feeder slots.

IV. Experimental Result Analysis

The algorithms proposed in this article are implemented in Python 3.8 by a desktop computer with Intel Core i7 1.8-GHz CPU and compared with aggregation mixed-integer programming (AMIP) [6], hybrid genetic algorithm (HGA) [9], cell division genetic algorithm (CDGA) [18], and optimizer integrated with an industrial software (ISO). Both HGA and CDGA are representatives of evolutionary algorithms for assembly optimization. AMIP, a mathematical programming technique combined with an aggregation technique, could optimize medium-sized

Algorithm 4: PAP Sequence Heuristic.

Input: PCB data with coordinate (X_p, Y_p) of point p, component assignment $\mathcal{C}$ and $\mathcal{K}$
Output: PAP sequence $\mathcal{P}$

Initialize $B = \{1, 2, \ldots, \beta\}$ as beam set where β is the beam width;
Initialize $\mathcal{P}, \mathcal{P}_b$ as empty matrix and $\mathcal{T}_b$ as $1 \times |H|$ matrix, $\forall b \in B$;
for $\mathcal{H}^{\text{CP}} \in \mathcal{C}, k \in \mathcal{K}$ **do**
  while $k \neq 0$ **do**
    Initialize $\beta \times 2$ matrix $\mathcal{W}$ as the coordinates of the β leftmost unplaced points;
    for $h \in H$ **do**
      Select β points which nearest to $\mathcal{W}(b), \forall b \in B$ with component type $\mathcal{H}^{\text{CP}}(h)$;
      Select β points among β^2 candidates with minimal Chebyshev distance as $p_1, \cdots, p_b$;
    end
    $k \leftarrow k-1, \mathcal{W}_b \leftarrow [X_{p_b}, Y_{p_b} - (h-1) \cdot \rho]$, $\mathcal{T}_b(h) \leftarrow p_b, \forall b \in B$;
    PAP sequence schedule for $\mathcal{T}_b$ using dynamic programming and attach $\mathcal{T}_b$ to $\mathcal{P}_b$ with column direction, $\forall b \in B$;
  end
end
$\mathcal{P} \leftarrow \mathcal{P}_b$ with minimal Chebyshev distance $\forall b \in B$;

TABLE II
Comparison of the Proposed Algorithms and the MIP Model

	Scale	Objective value			Computation time	
PCB	(N, C, P)	T_{scan}	T_{mip}	Gap (%)	t_{scan}	t_{mip}
1-1	$(1, 1, 14)$	4.735	4.408	7.42	0.29	323.60
1-2	$(2, 1, 14)$	4.314	3.833	12.55	0.34	34.03
1-3	$(3, 2, 16)$	4.095	3.886	5.83	0.20	984.10
1-4	$(4, 2, 20)$	4.720	4.165	13.33	0.27	1117.84
1-5	$(5, 3, 2)$	5.793	5.170	12.05	0.48	718.44
1-6	$(6, 3, 26)$	6.257	5.773	8.38	0.59	5445.63
AVG				9.93		

data in an acceptable amount of time. All the mathematical models mentioned in this article are solved using the optimizer Gurobi [24].

First, we compare the proposed algorithm to the optimal solution of the mixed-integer model, as shown in Table II. Based on the production result, the coefficients t_c, t_n, t_p, and t_m of the MIP model are set to 2, 6, 1, and 0.1, respectively. As the size of the problem increases, the model becomes less capable of solving the small-scale data in Table II. However, the solving efficiency of the proposed heuristic algorithms is substantially better than mathematical planning methods with an optimality gap of 9.93% average.

Second, we use several industrial PCB data, including a randomly generated complex one as representatives, to compare the result of different methods. The latter can be equated to a multibatch PCB assembly scenario without feeder setup change.

TABLE III
PCB DATA PARAMETERS

PCB	2-1	2-2	2-3	2-4	2-5	2-6	2-7	2-8
N	1	1	2	3	3	3	3	4
C	7	18	6	7	16	20	24	41
P	564	176	72	192	114	150	236	1510

TABLE IV
SUBOBJECTIVE COMPARISON

PCB	ISO	AMIP	HGA	CDGA	OUR
2-1	94,0,444[1]	95,0,490	420,0,444	94,0,432	95,0,490
2-2	30,0,56	30,0,115	36,0,54	40,0,86	30,0,52
2-3	16,0,22	16,0,48	16,0,16	16,0,24	16,0,22
2-4	32,1,74	38,0,122	64,0,80	48,0,80	32,1,64
2-5	20,0,37	20,0,78	24,0,30	24,0,30	20,0,30
2-6	26,2,98	32,2,94	33,0,108	81,3,84	32,0,96
2-7	42,1,68	–	46,0,62	44,4,102	45,0,64
2-8	290,9,552	–	370,0,425	280,9,812	288,2,440

[1] The comma-separated values represent the subobjectives of the number of cycles, nozzle changes, and pickups, respectively.

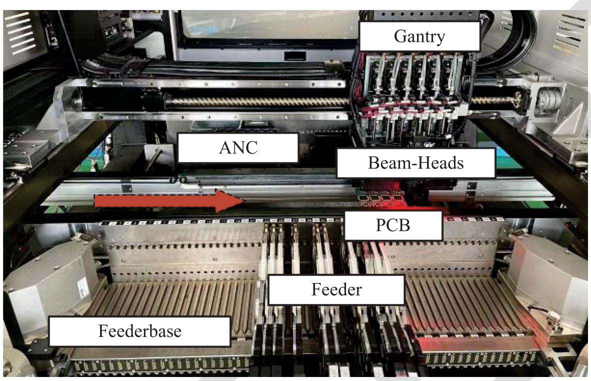

Fig. 4. Experimental platform of the placement machine.

TABLE V
CPH FOR DIFFERENT METHODS

	OUR	ISO		AMIP		HGA		CDGA	
PCB	E^1	E^2	ΔE^2	E^3	ΔE^3	E^4	ΔE^4	E^5	ΔE^5
2-1	11 297	11 255	0.37	6991	61.60	7035	60.35	10 673	5.84
2-2	16 058	16 003	0.35	11 460	40.21	14 958	7.36	12 462	28.86
2-3	12 451	11 998	3.78	9231	34.88	12 191	2.13	11 759	5.88
2-4	13 658	12 869	6.13	11 404	19.76	9795	39.43	10 423	31.03
2-5	13 375	13 022	2.72	9932	34.67	12 346	8.34	11 372	17.62
2-6	12 903	11 627	10.97	8843	45.91	10 457	23.39	7556	70.76
2-7	13 043	12 087	7.92	–	–	12 087	7.92	9830	32.69
2-8	13 557	11 835	14.55	–	–	11 781	15.08	10 477	29.40
AVG			5.85		39.49		20.50		27.76

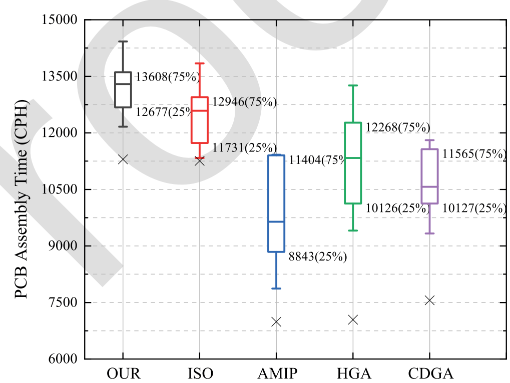

Fig. 5. Mounting time (CPH) distribution.

TABLE VI
TIME CONSUMING OF DIFFERENT METHODS

PCB	AMIP	HGA	CDGA	OUR
2-1	1.54	646.96	221.27	3.93
2-2	0.83	159.27	23.61	2.31
2-3	0.66	29.93	4.37	0.73
2-4	1.26	136.48	6.30	1.05
2-5	2.83	82.18	13.97	1.17
2-6	13.92	129.21	20.74	3.47
2-7	–	215.43	40.06	5.20
2-8	–	635.00	171.89	23.25
AVG	–	94.21	204.65	11.93

The comparative PCB data parameters are shown in Table III. According to the machine parameters, we set $e = 0.5$, $e_1 = 4$, and $e_2 = 0.6$ in the implementation of the heuristic algorithms. We set the size of the beam in the beam search to half the number of placement heads. This research investigates the effects of the optimization technique without feeder prearrangement since AMIP, HGA, and CDGA cannot deal with prearrangement conditions, and AMIP and HGA can only optimize single feeder type. The experiment findings indicate the suggested approach, ISO, AMIP, HGA, and CDGA, respectively, as $E^i(i = 1, 2, 3, 4, 5)$. The performance improvement of the suggested approach over other methods is represented by ΔE^i, which is computed as $\Delta E^i = (E^1 - E^i)/E^1 \times 100\%, i = 2, 3, 4, 5$.

This article compares the main subobjective values of optimization method results with each other, as shown in Table IV. The number of PAP cycles is one of the overall performance subobjectives since, in some cases, it may affect the distance of the moving route. The method proposed in this article exhibits more effective search capabilities when dealing with complex data.

Algorithm verification is done on our placement machine platform, which is shown in Fig. 4. We convert the assembly time into the standard time chip per hour (CPH) to provide a straightforward comparison independent of the number of placement points. A batch of PCBs is subjected to each procedure three times, and Table V shows the average assembly time. Even though the proposed algorithm does not significantly outperform the industrial customize optimizer results for small- and medium-sized data, its advantages become more evident as the size of the problem increases. The assembly efficiency distribution shown in Fig. 5 shows that the proposed algorithm is more stable than others.

The search efficiency is compared with other methods in Table V except for the built-in industrial customize optimizer. It can be seen that evolutionary-based algorithms take a longer time to find a solution, and the results are usually unstable due to their random exploration. AMIP is still intractable for large-scale PCB data, despite the efficient aggregate-based technique incorporated.

TABLE VII
CPH FOR DIFFERENT METHODS WITH MULTIWIDTH FEEDERS

PCB	Parameter P	C	N	Objective value E^1	E^2	ΔE^2
3-1	78	16	3	10 912	10 688	2.06
3-2	150	20	3	8493	8229	3.11
3-3	110	23	3	13 001	12 811	1.46
3-4	161	38	3	11 143	8798	21.04
3-5	540	10	4	8416	7548	10.31
AVG						7.60

The feeder allocation has a pivotal impact on the overall assembly efficiency, but only some researchers elaborate on the solution to the feeder types with different widths. We conduct comparative tests with PCB data using different width feeders to compare the suggested approach with the ISO method. According to Table VII, the proposed method provides a 7.60% overall efficiency gain over the industrial customize optimizer.

V. CONCLUSION

The scan-based hierarchical heuristic algorithm demonstrated excellent performance and efficient search in solving the complex surface mount optimization problem. We proposed a mixed integer mathematical model and elaborately designed heuristic algorithms. The component pickup procedure inspired the techniques of feeder allocation and component assignment with linear-aligned heads. While the component assignment heuristic algorithm concentrated on multihead pickup, the heuristic feeder allocation approach emphasized feeder allocation, increasing simultaneous pickup numbers. The ultimate goals of both the algorithms were to improve pickup efficiency and decrease nozzle change. In this article, beam search was used to improve the search quality of the PAP route schedule. In terms of extension, the algorithm analyzed the requirements in various application scenarios and gave supporting solutions to be indeed applied to industrial production environments. The experiments compared several previous research and an industrial optimizer, and the findings demonstrated that the suggested technique considerably increased the efficiency of placement machine assembly.

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A Scan-Based Hierarchical Heuristic Optimization Algorithm for PCB Assembly Process

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***Abstract*—Surface mount technology is essential to the development of the electronic manufacturing industry. This article studies optimizing the surface mount process for the beam-head placement machine. A mixed-integer programming (MIP) model is proposed for this problem, which is decomposed into three interconnected hierarchical parts: feeder allocation; component assignment; and pick-and-place (PAP) sequence problems. This article proposes an efficient hierarchical framework with three elaborately designed heuristics to solve the above problem. The design of the scan-based algorithms optimizes the subobjectives of feeder allocation and component assignment. First, the allocation heuristic arranges the feeders into slots as a prerequisite for other problems. Then, the component assignment heuristic determines the component type for each head with a variety of criteria and long short-term objectives. Finally, the PAP sequence problem is solved using a modified beam search algorithm. The proposed algorithm offers advantages in terms of effectiveness, efficiency, and extension, which can satisfy various customization demands. Experiments are conducted on our self-designed placement machine using industrial and randomly generated data. Computational experiments show that the scan-based heuristic algorithm obtains near-optimal solutions with a gap of 9.93% averagely compared with the proposed MIP model and provides efficiency improvement over the mainstream studies.**

***Index Terms*—Hierarchical decomposition, mixed-integer linear model, printed circuit board (PCB) assembly optimization, scan-based heuristic.**

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I. Introduction

NOWADAYS, the widespread use of electronic products in modern life has raised attention to the price and quality of printed circuit boards (PCBs). A complicated collection of production procedures makes up for manufacturing electronic products. PCB assembly is one of the necessary but time-consuming processes among them. The placement machine is a sophisticated computer-controlled apparatus that integrates mechanical, electrical, and optical techniques [1]. The factory uses automatic manufacturing lines to produce high-quality PCB, and the maximum production capacity of the placement machine is the efficiency bottleneck of the whole production line. The application benefit of an effective assembly optimization technique is enormous.

This article focuses on the beam-head placement machine, which has a stationary PCB platform, two stationary feeder bases, and a moving gantry with beam heads, as shown in Fig. 1. The feeders loading with components are installed on the feeder base. There are three basic types of feeders for assembling various package component parts: tape, stick, and tray. The gantry moves between the PCB and the feeder base to pick and place the components with vacuum valves. The fly camera is equipped in the heads for chip detection; for some large chips, the gantry moves to the fixed camera for inspection. An auto nozzle changer (ANC) is kept with multiple nozzle types to fulfill the assembly needs for various component shapes. The primary distinction between the beam-head placement machine and other types is its mechanical design, which enables multiheads to pick up components from feeders simultaneously.

As shown in Fig. 2, the surface mount process consists of six different types of operations, and the dashed line framed part includes a pick-and-place (PAP) cycle, which is the fundamental unit. The nozzle change, component pickup, and component placement operations in a PAP cycle take substantial time, and

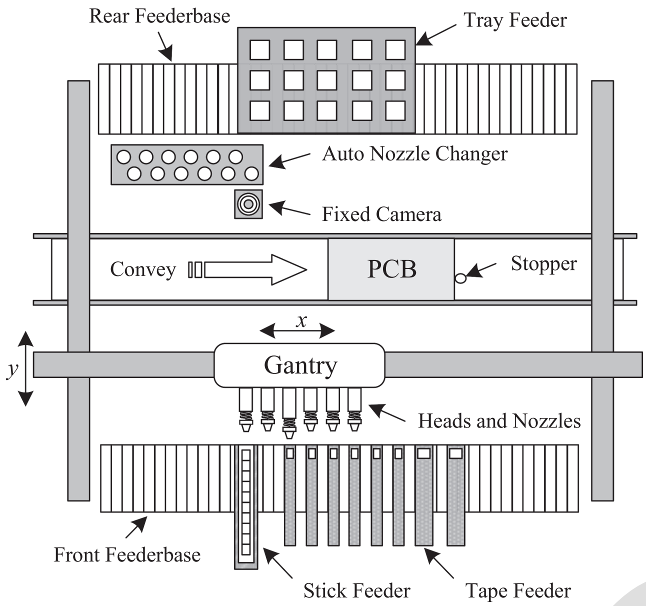

Fig. 1. Layout of the beam-head placement machine.

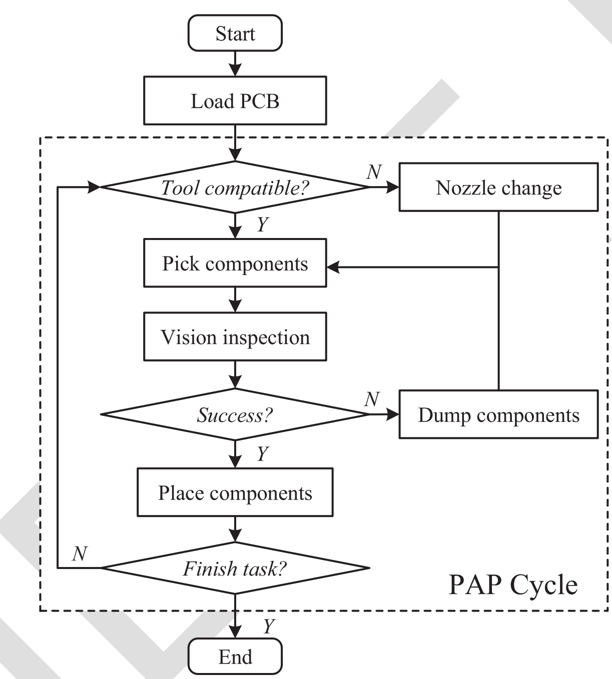

Fig. 2. Workflow chart of the surface mount process.

the algorithm can optimize the first two operations. More specifically, by combining multiple head motions, the pickup operation could be more effective, and the nozzle changes are connected to the sequence of component pickup. The component dumping operations caused by image processing errors are exceptions and are not considered in this article.

The surface mount optimization problem has multiple variables with significant coupling and is an NP-hard problem [2]. A general technique for this complex optimization problem is the hierarchical decomposition method [3], [4]. This challenging combinatorial optimization problem is made up of the location problem, the assignment problem, and the route schedule problem. In the locating problem, the depots are feeders assigned for the assembling process [5]. The assignment problems are concerned with determining the type of component picked up by the placement head, which must take into account the tool compatibility of the nozzle component [6], [7] and its influence on simultaneous pickup [8], [9], both of which are essential factors influencing assembly efficiency. The PAP route schedule is covered in studies [10] and [11] utilizing heuristic and mathematical programming, respectively.

Surface mount optimization has been solved by a variety of algorithms, such as mathematical programming, evolutionary algorithms, tailored constructive heuristics, etc. The mathematical programming method is limited by the complexity of the problem, and the subproblem is the subject of multiple studies [12], [13], [14]. Medium-size problems can be solved using mathematical programming combined with the aggregation [6] approach and the augmented technique for multiobjective [15]. Evolutionary algorithms have been widely used in surface mount optimization problems [16], [17], [18], [19], [20], such as the genetic algorithm, particle swarm algorithm, shuffle leapfrog method, etc. Complex optimization problems may have multiple subobjectives, and research has been done to combine multiobjective optimization with evolutionary algorithms to find the Pareto fronts of the problems [21], [22]. Some studies provide constructive heuristics, which solve the problem based on the structure of the problem and significantly improve the quality of solutions [23].

To summarize, there is still a long way to industrial deployments. Current research is flawed by irrational assumptions or inadequate examination of the factors influencing assembly efficiency. The algorithm needs to be constructed to work in various application scenarios. In this article, we propose a mathematical model and a novel hierarchical scan-based heuristic framework for the surface mount optimization problem. The contributions of this article are summarized as follows.

1) A mixed-integer model for the PCB assembly process is proposed. The model fully incorporates the factors affecting assembly efficiency and decomposes the assembly process into pickup and placement parts. The pickup model takes into account the impact of simultaneous pickup on efficiency for the first time, and the placement model is modeled as a variant of the multiple traveling salesman problem (MTSP).
2) The hierarchical decomposition approach reduces the complexity of the problem. Based on the problem characteristics for each subproblem, three elaborately designed heuristics combined with the scanning concept are proposed, which can obtain a nearly optimal solution and perform better on the search efficiency compared to other approaches.
3) The proposed algorithms demonstrate substantial extensions, which are adaptable enough to satisfy the operators' various customized requirements. The algorithm optimization process simulates the pickup process, which can

be adapted to the actual situation of the feeder allocation and component pickup operation tasks.

The rest of this article is organized as follows. Section II presents a mixed-integer mathematical model, and Section III proposes a scan-based hierarchical heuristic algorithm to provide a satisfying PCB assembly solution. In Section IV, the experiment results are introduced and compared with the mainstreaming study. Finally, Section V concludes this article.

II. Mathematical Model

A. Problem Description

The surface mount optimization is to solve the scheduling problem of the PCB assembly process and get an efficient solution with complicated constraints and multiple decision variables. The typical subproblems of the assembly process are the feeder allocation problem and the head task assignment problem. The former solves the problem of the arranged slots of feeders, while the latter determines the assembly sequence. The component assignment problem and the PAP route schedule problem are further decompositions in this article for the head task assignment. There is a progressive relationship between the two subproblems, and the complexity of the problem can be reduced by determining the component type and then the placement point of each head.

The underlying subproblems are tightly coupled. The feeder allocation affects component assignment for maximizing the number of simultaneous pickups, i.e., combining more pickup operations. The pickup slots of the assembly process and the assembly sequence determine the overall movement distance of the gantry. There may be redundant movements for pickup operations and nozzle changes for the consistency of the nozzle type, component type, and feeder slot.

This article makes the following assumptions about the optimization problem with litter impact on the optimality of the solution.

1) The X- and Y-axis motor movement is simplified to an independently controlled trapezoidal profile.
2) The interval distance between adjacent heads is integer times the interval distance between two adjacent feeder slots.
3) Only an appropriate type of nozzle can pick up the component.
4) The ANC configuration is predetermined, and the movement at different holes is ignored.
5) Tray and stick feeders have predetermined arrangements and are not incorporated into the optimization process.

B. Optimization Objective and Constraints

The surface mount process is accomplished by a complex series of motions that work together. The target of minimizing the assembly time depends on the distance of the gantry traveling, the number of pickup operations, and the number of nozzle change operations, which are the subobjectives. The coupling of subobjectives is reflected in combining the pickups of multiple heads, which may bring additional nozzle change, and the distance of the gantry traveling relies on the pickup and nozzle change operations.

The constraints for surface mount optimization problems can be divided into four categories: job completion constraint, mechanical restriction, tool requirement, and artificial constraints. Job completion is essential for surface mount tasks, and each component must be assembled accurately on the corresponding PCB pads. The mechanical restriction concerns the structural characteristics of the placement machine, such as each head having unreachable pickup slots. Another type of mechanical constraint is positional interference caused by feeders occupying multiple slots. The restricted number of nozzles and feeders available will also impede optimizing assembly efficiency. Tool consistency is a critical assurance for the assembly process. In terms of artificial limits, operators may want to prearrange feeders, prohibit some feeder slots, and set prohibited heads.

C. Mixed-Integer Programming (MIP) Model

Mathematical programming methods to solve the surface placement task must deal with the problems of numerous decision variables and intricate constraints. The route scheduling of the gantry is constrained by the type of component, nozzle, and slot that corresponds to each head, which greatly increases the complexity of the model. This article proposes a hierarchical MIP model to solve the problem effectively by decomposing the surface mount process into two parts: the pickup model and the placement model. The pickup model is a prerequisite for the solution of the placement model, which determines the movement time parameters and the placement head task in the placement model. The notations of the proposed model are shown in Table I. The table describes the type of decision variables, all of which are nonnegative.

1) Pickup Model:

$$\min t_c \cdot \sum_{k\in K} g_k + t_n \cdot \sum_{h\in H} d_h + t_p \cdot \sum_{s\in S'} \sum_{k\in K} e_{sk} + t_m \cdot \sum_{k\in K} u_k \tag{1}$$

$$g_k \geq g_{k+1} \quad \forall k \in K \backslash \{|K|\} \tag{2}$$

$$\sum_{i\in I} \sum_{s\in S} x_{iskh} \leq g_k \quad \forall k \in K, h \in H \tag{3}$$

$$\sum_{j\in J} \sum_{i\in I} \sum_{s\in S} \mu_{ij} \cdot x_{iskh} \leq 1 \quad \forall k \in K, h \in H \tag{4}$$

$$\sum_{s\in S} \sum_{h\in H} \sum_{k\in K} x_{iskh} = \psi_i \quad \forall i \in I \tag{5}$$

$$d_h = \frac{1}{2} \sum_{k\in K\backslash\{|K|\}} \left(\sum_{j\in J} \left| \sum_{i\in I} \sum_{s\in S} \mu_{ij} \cdot x_{iskh} - \sum_{i\in I} \sum_{s\in S} \mu_{ij} \cdot x_{is(k+1)h} \right| - 1 \right) \quad \forall h \in H \tag{6}$$

$$e_{sk} \leq \sum_{i\in I} \sum_{h\in H_s} x_{i[s+(h-1)\cdot\tau]kh} \leq M \cdot e_{sk} \quad \forall s \in S', k \in K \tag{7}$$

TABLE I
SUMMARY OF NOTATIONS

	Notation	Description
Indexes & Sets	$i \in I$	Index of component type, $I = \{1, 2, \cdots\}$
	$j \in J$	Index of nozzle type, $J = \{1, 2, \cdots\}$
	$p, q \in P$	Index of (placement) point, $P = \{1, 2, \cdots\}$
	$h, l \in H$	Index of head, $H = \{1, 2, \cdots\}$[1]
	$a \in A$	Index of arc, $A = \{(h, l) \mid h \neq l, h, l \in H\}$[2]
	$k \in K, K'$	Index of cycle, $K = \{1, 2, \cdots\}, K' = \{k \mid g_k > 0\}$
	$s, r \in S$	Index of slot, $S = \{1, 2, \cdots\}$[3]
Parameters	t_c	Average moving time of round trip between PCB and feeder base
	t_n	Average time of nozzle change operation
	t_p	Average time of pickup operation
	t_m	Average moving time on the feeder base per slot
	μ_{ij}	Compatibility of component type i and nozzle type j
	η_{ip}	Correspondence of component type i and point p
	ψ_i	Number of points of component type i
	ϕ_i	Feeder number of component type i available
	ζ_j	Number of nozzle type j available
	λ^{FW}_{pkh}	Moving time from feeder base to the first point in cycle k
	λ^{PL}_{pqa}	Moving time between point p and point q along with arc a
	λ^{BW}_{pkh}	Moving time from the last point to feeder base in cycle k
	ρ	Interval distance between adjacent heads
	τ	Interval ratio of adjacent heads and adjacent slots
	M	Sufficiently large number
Decision Variables[4]	g_k	Binary variable = 1 iff at least one point is picked up and placed in cycle k
	u_k	Integer variable the pick-up moving slot of cycle k
	d_h	Integer variable the number of nozzle changes of head h
	e_{sk}	Binary variable = 1 iff component is picked up from the equality slot s in cycle k
	f_{si}	Binary variable = 1 iff comp. type i is assigned to slot s
	x_{iskh}	Binary variable = 1 iff head h picks up component type i from slot s in cycle k
	w_{pqka}	Binary variable = 1 iff point p is placed after point q along with arc a in cycle k
	y_{pkh}	Binary variable = 1 iff point p is the first point placed with head h in cycle k
	z_{pkh}	Binary variable = 1 iff point p is the last point placed with head h in cycle k

[1] The head set H_s is the subset H, which contains the heads that can pick up components from slot s, $H_s = \{\max(1, -\lfloor (s-1)/\tau \rfloor + 1), \cdots, \min(|H|, \lceil (|S| - s + 1)/\tau \rceil) + 1\}$

[2] The arcs of A represent the placement sequence of the heads. A_h, A_h^f and A_h^t are subsets of A, where A_h has the arcs of A that pass head h, A_h^f has the arcs of A from head h, and A_h^t has the arcs of A to head h.

[3] S' is the set of equality slot index, which refers to the left-most head aligned slot, $S' = \{-\tau \cdot (|H| - 1) + 1, \cdots, 1, 2, \cdots, |S|\}$

[4] The intermediate continuous variables v_{pq}, n_p, and m_p are used to eliminate subtour.

$$u_k \geq s \cdot e_{sk} - r \cdot e_{rk} \quad \forall k \in K, s, r \in S' \tag{8}$$

$$f_{si} \leq \sum_{h \in H} \sum_{k \in K} x_{iskh} \leq M \cdot f_{si} \quad \forall s \in S, i \in I \tag{9}$$

$$\sum_{i \in I} f_{si} \leq 1 \quad \forall s \in S \tag{10}$$

$$\sum_{h \in H} \sum_{i \in I} \sum_{s \in S} \mu_{ij} \cdot x_{iskh} \leq \zeta_j \quad \forall k \in K, j \in J \tag{11}$$

$$\sum_{s \in S} f_{si} \leq \phi_i \quad \forall i \in I. \tag{12}$$

The objective of the pickup model (1) consists of four terms: the number of cycles; nozzle change operations; pickup operations; and pickup moving distance; where the pickup moving distance is represented by the number of slots the gantry crosses over. Constraint (2) ensures that the first few cycles of the surface mount process are given top priory for completion. The heads and work cycle are consistent with Constraint (3). Constraint (4) ensures that each head is equipped with at most one nozzle type. The completion of the surface mount process for each component type is guaranteed by constraint (5). Constraint (6) calculates the number of nozzle changes of each head, and constraint (7) converts the pickup slot of each head to the leftmost head to calculate the number of the pickup operations in each cycle. Constraint (8) calculates the number of slots crossed over by the gantry for the pickup process in each cycle. Constraint (9) ensures the consistency of head pickup operations and feeder allocation. Constraint (10) ensures that each slot is assigned at most one feeder. Constraints (11) and (12) indicate the limited number of available nozzles and feeder base, respectively.

2) Placement Model:

$$\min \sum_{k \in K'} \left\{ \sum_{p \in P} \sum_{h \in H} \lambda^{FW}_{pkh} \cdot y_{pkh} + \sum_{p \in P} \sum_{q \in P} \sum_{a \in A} \lambda^{PL}_{pqa} \cdot w_{pqka} + \sum_{p \in P} \sum_{h \in H} \lambda^{BW}_{pkh} \cdot z_{pkh} \right\} \tag{13}$$

$$\sum_{q \in P} \sum_{a \in A_h} w_{pqka} = \sum_{i \in I} \sum_{s \in S} \eta_{ip} \cdot x_{iskh} \quad \forall p \in P, k \in K', h \in H \tag{14}$$

$$\sum_{p \in P} \sum_{q \in P} \sum_{a \in A_h} w_{pqka} \leq 2 \quad \forall k \in K', h \in H \tag{15}$$

$$\sum_{p \in P} (y_{pkh} + z_{pkh}) \leq 1 \quad \forall k \in K', h \in H \tag{16}$$

$$\sum_{q \in P} \sum_{a \in A_h^t} w_{qpka} + y_{pkh} = \sum_{q \in P} \sum_{a \in A_h^f} w_{pqka} + z_{pkh} \quad \forall k \in K', h \in H, p \in P \tag{17}$$

$$y_{pkh} \leq \sum_{q \in P} \sum_{a \in A_h^f} w_{pqka} \quad \forall k \in K', h \in H, p \in P \tag{18}$$

$$z_{pkh} \leq \sum_{q \in P} \sum_{a \in A_h^t} w_{qpka} \quad \forall k \in K', h \in H, p \in P \tag{19}$$

$$\sum_{p \in P} \sum_{h \in H} y_{pkh} = 1 \quad \forall k \in K' \tag{20}$$

$$\sum_{p \in P} \sum_{h \in H} z_{pkh} = 1 \quad \forall k \in K' \tag{21}$$

$$\sum_{k \in K'} \left(\sum_{h \in H} y_{pkh} + \sum_{q \in P} \sum_{a \in A} w_{pqka} \right) = 1 \quad \forall p \in P \tag{22}$$

$$\sum_{k\in K'}\left(\sum_{h\in H} z_{pkh} + \sum_{q\in P}\sum_{a\in A} w_{qpka}\right) = 1 \quad \forall p \in P \qquad (23)$$

$$m_p + \sum_{q\in P} v_{pq} - n_p - \sum_{q\in P} v_{qp} = 1 \quad \forall p \in P \qquad (24)$$

$$v_{pq} \le \sum_{k\in K'}\sum_{a\in A} (|P| - |K'| + 1)\cdot w_{pqka} \quad \forall p, q \in P \qquad (25)$$

$$n_p \le \sum_{k\in K'}\sum_{h\in H} (|P| - |K'| + 1)\cdot y_{pkh} \quad \forall p \in P \qquad (26)$$

$$m_p \le \sum_{k\in K'}\sum_{h\in H} (|P| - |K'| + 1)\cdot z_{pkh} \quad \forall p \in P. \qquad (27)$$

The objective of placement model (13) is the total of the moving times except for the pickup movement, which has been solved in the pickup model. The parameters of moving time λ in the objective are obtained based on the solution of the pickup model. Constraint (14) ensures that the solutions of the pickup model and the placement model are consistent. Constraints (15) and (16) ensure that each head is placed at most one placement point. Constraints (17)–(19) ensure the continuity of the placement task, i.e., the placement head is unique for each point. Constraints (20) and (21) mean that the path of the placement head from the feeder base to the PCB and from the PCB back to the feeder base is unique for each cycle. Constraints (22) and (23) ensure that the entry edge and the leave edge of each point are unique, respectively. Constraints (24)–(27) are utilized to eliminate the subtour for each cycle.

The pickup model (1)–(12) and placement model (13)–(27) involve an assignment problem and a restricted MTSP problem, which are two well-known NP-hard problems. Therefore, the proposed model above can be solved only for small-scale data in a reasonable amount of time. In Section III, we will further decompose the problem following the optimization objective, and an efficient hierarchical framework will be proposed to solve this problem.

III. Hierarchical Heuristic Optimization

A. Scan-Based Heuristic Hierarchical Framework

Hierarchical decomposition is a common method for solving complicated optimization problems. A direct solution to the whole problem may bring on a dimensionality disaster because of the numerous constraints and decision variables. It makes sense to design the algorithm by the relevance of the subobjective. The constructive scan heuristic algorithm [5] is the basis of the proposed method in this article, which overcomes the shortcomings of the lengthy solving time and greedily maximizes the pickup efficiency.

This article decomposes the surface mount optimization problem into the feeder allocation problem, component assignment problem, and PAP sequence problem. We prioritize feeders since ignoring them will significantly increase pickup operations and longer moving routes. Furthermore, if the feeder arrangement must be changed each time the PCB changes, the labor cost

Algorithm 1: Feeder Allocation Heuristic.

Input: PCB data, component data, feeder data, and nozzle pattern $\mathcal{N}$
Output: feeder assignment $\mathcal{F}^{\mathrm{CP}}$ and $\mathcal{F}^{\mathrm{PT}}$
Initialize $\mathcal{F}^{\mathrm{CP}}$ as the component type prearranged on the feeder base (-1 for empty), $\mathcal{F}^{\mathrm{PT}}$ as the number of the placement points, and $\mathcal{S}$ as an empty stack;
while $\sum_{i\in I}\psi_i \neq 0$ **do**
  Initialize $V_b \leftarrow 0$ as the best allocation value;
  for $s \leftarrow 1$ to $|S| - (|H| - 1)\cdot\tau$ **do**
    foreach $s' = s + (h-1)\cdot\tau, h \in H$ **do** $\mathcal{H}^{\mathrm{CP}}(h) \leftarrow \mathcal{F}^{\mathrm{CP}}(s'), \mathcal{H}^{\mathrm{PT}}(h) \leftarrow \mathcal{F}^{\mathrm{PT}}(s')$
    $I' \leftarrow I$;
    for $j \leftarrow \mathcal{N}(h), h \in \{h' | \mathcal{H}^{\mathrm{CP}}(h') > 0\}$ **do**
      if $\psi_i = 0, \forall i \in \{i' | \xi_{i'j} \neq 0, i' \in I'\}$ **then**
        push $i \leftarrow \mathrm{argmax}_{i'\in I'}\{\psi_{i'}\}$ into $\mathcal{S}$;
      else
        $i \leftarrow \mathrm{argmax}_{i'\in I'}\{\psi_{i'} | j\cdot\xi_{i'j} > 0\}$
        $\mathcal{H}^{\mathrm{CP}}(h) \leftarrow i, \mathcal{H}^{\mathrm{PT}}(h) \leftarrow \psi_i$;
      end
      $I' \leftarrow I' \backslash \{i\}$;
    end
    Pop components from $\mathcal{S}$ and assign them to the heads $h \in \{h' | \mathcal{H}^{\mathrm{PT}}(h') = 0\}$;
    if $\sum_{h\in H}\mathcal{H}^{\mathrm{PT}}(h) > V_b$ **then**
      $V_b \leftarrow \sum_{h\in H}\mathcal{H}^{\mathrm{PT}}, \mathcal{H}_b^{\mathrm{PT}} \leftarrow \mathcal{H}^{\mathrm{PT}}, \mathcal{H}_b^{\mathrm{CP}} \leftarrow \mathcal{H}^{\mathrm{CP}}$, $s_b \leftarrow s$
    end
  end
  $\delta = \min\{\mathcal{H}_b^{\mathrm{PT}}(h) | \mathcal{H}_b^{\mathrm{PT}}(h) \neq 0, h \in H\}$;
  for $s' \leftarrow s_b + (h-1)\cdot\tau, h \in H$ **do**
    if $\mathcal{F}^{\mathrm{PT}}(s') = -1$ **then**
      $\mathcal{F}^{\mathrm{CP}}(s') \leftarrow \mathcal{H}_b^{\mathrm{CP}}(h)$;
    end
    $\mathcal{F}^{\mathrm{PT}}(s') \leftarrow \mathcal{F}_b^{\mathrm{PT}}(s') - \delta$;
    $\mathcal{N}(h) \leftarrow j, \psi_i \leftarrow \psi_i - \delta$ where $i = \mathcal{H}_b^{\mathrm{CP}}(h)$, $j = \sum_{j'\in J} j'\cdot\xi_{ij'}$;
  end
end

associated with reoptimizing the algorithm could increase. The PAP route schedule is the final subproblem to be solved since the moving distance of the placement heads has less impact on assembly efficiency than other factors.

The relationship among subproblems, subobjectives, and constraints is shown in Fig. 3. The feeder allocation and component assignment problems impact the nozzle changes and simultaneous pickups, while the route schedule problem is relatively independent. It can be expected that there are certain similarities in the algorithm design of feeder allocation and component assignment. The superscripts NZ, CP, and PT of the notations in the algorithm description are the abbreviation of nozzle type, component type, and the number of placement points, respectively.

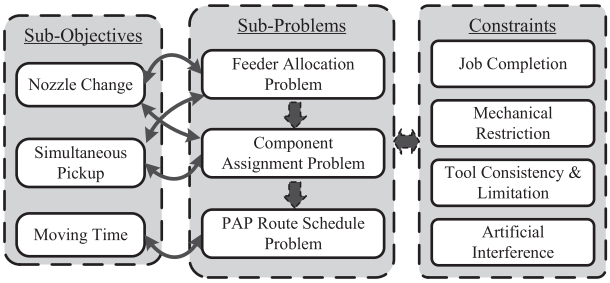

Fig. 3. Relationship of surface mount process optimization subobjectives, subproblems, and constraints.

B. Feeder Allocation Heuristic Algorithm

Feeder allocation is a prerequisite for other subproblems, and an appropriate arrangement will significantly enhance pickup efficiency, which determines the component pickup slot. The basic idea of feeder allocation heuristic described in Algorithm 1 is assigning the feeders while scanning the feeder base under the constraint of the nozzle pattern, which can maximize the number of pickup points allocated in a round and avoid nozzle change. The algorithm assigns feeders to the empty slots in the different rounds, reserving the component types already arranged in the head-aligned slots. The component types that can be allocated in the head-aligned slots are determined by the nozzle pattern. The nozzle pattern helps to reduce the number of nozzle changes for subsequent pickup operations. The type of component with more placement points that do not meet the nozzle pattern restriction is stored in component stacks to guarantee a comparably concentrated position of the feeder allocation. At the end of the assignment, the algorithm assigns components in the stack to slots.

C. Component Assignment Heuristic Algorithm

The algorithmic framework for feeder allocation and component assignment is similar, and both are based on heuristic scanning. The feeder allocation solves the problem of component pickup position, and the component assignment solves the problem of pickup sequence. The scanning heuristic efficiently optimizes the simultaneous pickups, which significantly reduces the overall pickup operations by integrating the pickup operations of multiheads. Similar to feeder allocation produces, each head aligns to a slot starting from different pickup slots, the component assigned to the head should satisfy the following criteria.

1) *Pickup feasibility:* The head-aligned slot contains unpicked placement points.
2) *Pickup constraint:* The head-equipped number of nozzles does not exceed the number available.
3) *Pickup prejudgment:* The component being picked up does not lessen the number of subsequent simultaneous pickups of the prejudgment.
4) *Pickup objective:* The efficiency gain from pickup outweighs the efficiency loss from nozzle change.

Algorithm 2: Component Assignment Heuristic.

Input: PCB data, feeder allocation $\mathcal{F}^{\mathrm{CP}}$ and $\mathcal{F}^{\mathrm{PT}}$
Output: component assignment $\mathcal{C}$ and cycle group $\mathcal{K}$
Initialize a $1 \times |H|$ matrix $\mathcal{M}$ of *None* as the initial nozzle assignment;
while $\sum_{s \in S} \mathcal{F}^{\mathrm{PT}}(s) \neq 0$ **do**
  Initialize $V_b \leftarrow 0$ as the best assignment value;
  for $\mathcal{N} \in \mathcal{M}$, $s \leftarrow 1$ to $|S| - (|H| - 1)\tau$ **do**
    for $h \in H$ **do**
      $s' \leftarrow s + (h-1)\tau$, $i \leftarrow \mathcal{F}^{\mathrm{CP}}(s')$;
      Calculate $v \leftarrow e_1 \cdot v_1 - e_2 \cdot v_2$ where $v_1 = \min_{h' \in H}\{\mathcal{H}^{\mathrm{PT}}(h') > 0\} \cup \{\mathcal{F}^{\mathrm{PT}}(s')\}$, $v_2 = \sum_{h' \in H} |\mathcal{N}(h') - \sum_j \xi_{\mathcal{H}^{\mathrm{PT}}(h') \cdot j}|$;
      if $\mathcal{F}^{\mathrm{PT}}(s') > 0$ **and** $v > 0$ **then**
        $\mathcal{H}^{\mathrm{CP}}(h) \leftarrow \mathcal{F}^{\mathrm{CP}}(s')$, $\mathcal{H}^{\mathrm{PT}}(h) \leftarrow \mathcal{F}^{\mathrm{PT}}(s')$;
      end
    end
    Calculate short-term objective V_s and long-term objective V_l with Algorithm 3;
    if $e \cdot V_l + (1-e) \cdot V_s > V_b$ **then**
      $V_b \leftarrow e \cdot V_l + (1-e) \cdot V_s, s_b \leftarrow s$;
      $(\mathcal{H}_b^{\mathrm{PT}}, \mathcal{H}_b^{\mathrm{CP}}, \mathcal{H}_b^{\mathrm{NZ}}) \leftarrow (\mathcal{H}^{\mathrm{PT}}, \mathcal{H}^{\mathrm{CP}}, \mathcal{H}^{\mathrm{NZ}})$
    end
  end
  $k \leftarrow \min_{h \in H}\{\mathcal{H}_b^{\mathrm{PT}}(h) > 0\}$;
  foreach $h \in H$ **do**
   $s' \leftarrow s_b + (h-1) \cdot \tau, \mathcal{F}^{\mathrm{PT}}(s') \leftarrow \mathcal{F}^{\mathrm{PT}}(s') - k$
  if $\mathcal{H}_b^{\mathrm{PT}}(h) > 0$ **or** $\mathcal{F}^{\mathrm{PT}}(s) = 0, \forall h \in H, s \in S$ **then**
    Attach $\mathcal{H}_b^{\mathrm{CP}}$ to $\mathcal{C}$, $\mathcal{H}_b^{\mathrm{NZ}}$ to $\mathcal{M}$, k to $\mathcal{K}$ along with column direction;
  end
end

Algorithm 2 describes the implementation of the component assignment. Each round determines the type of component assigned to heads with unpicked placement points and the related cycle groups. A "cycle group" is a set of consecutive PAP cycles with the same component assignments. It should be mentioned that the scanning-based pickup procedure tries to maximize the number of simultaneous pickups while minimizing the expense of nozzle changes. The component assignment heuristic is forward looking, which means that the single-head component assignment prejudges its impact on subsequent assignments. This is principally reflected in the following two aspects: the first is to assign just those components that improve the overall objective, and the second is the long short-term objectives. As for long short-term objectives implemented in Algorithm 3, the long-term objective is to simultaneously pick up components from all the aligned slots until one is empty, while the short-term goal is to pick up all the components from the aligned slots greedily. The current component assignment result is the short-term objective, and its effect on pickup efficiency as a whole is the long-term objective. The long short-term objective is the weighted sum of these two.

Algorithm 3: Long Short-Term Objective Calculation.

Input: Head component assignment $\mathcal{H}^{\mathrm{PT}}$
Output: short-term objective V_s and long-term objective V_l
Initialize short-term objective $V_s \leftarrow 0$ and long-term objective $V_l \leftarrow -e_2 \cdot \sigma$;
$V_s \leftarrow e_1 \cdot \omega \cdot \min_{h' \in H}\{\mathcal{H}^{\mathrm{PT}}(h') > 0\} - e_2 \cdot \sigma$ where $\omega = |H| - |\{h' | \mathcal{H}^{\mathrm{PT}}(h') > 0, h' \in H\}| - 1$ and $\sigma = \sum_{h' \in H} |\mathcal{N}(h') - \sum_{j \in J} j \cdot \xi_{\mathcal{H}^{\mathrm{CP}}(h') \cdot j}|$;
while $\mathcal{H}^{\mathrm{PT}}(h) > 0, \exists h \in H$ **do**
  $V_l \leftarrow V_l + e_1 \cdot \omega \cdot \min_{h' \in H}\{\mathcal{H}^{\mathrm{PT}}(h') > 0\}$ where $\omega \leftarrow |H| - |\{h' | \mathcal{H}^{\mathrm{PT}}(h') > 0, h' \in H\}| - 1$;
  $\mathcal{H}^{\mathrm{PT}} \leftarrow \mathcal{H}^{\mathrm{PT}} - \min_{h' \in H}\{\mathcal{H}^{\mathrm{PT}}(h') > 0\}$;
  foreach $h' \in H$ **do** $\mathcal{H}^{\mathrm{NZ}}(h') \leftarrow \sum_{j \in J} j \cdot \xi_{\mathcal{H}^{\mathrm{CP}}(h') \cdot j}$
end

D. PAP Sequence Heuristic Algorithm

The pick and placement route schedules make up the PAP route schedule problem. In case the feeder allocation and the component assignment are determined, the pickup procedure calls for picking up components from each preset slot in a single direction on the feeder base. Algorithm 4 shows the process of beam search, which is utilized to solve the placement route schedule problem by retaining multiple potentially optimal solutions based on greedy search. The placement process can be thought of as a constrained vehicle route schedule problem with capacity constraints and candidate placement point constraints imposed by the component assignment. The dynamic programming is employed to determine the placement sequence in each cycle, which is efficient with a limited number of placement points.

E. Extension of the Proposed Algorithm

The proposed algorithms show significant applicability expansion. First, the algorithm may balance the nozzle change and pickup operation cost by modifying the parameter weights. Second, regardless of the number of linear-aligned heads, the technique may be utilized to achieve simultaneous pickup. Even though the adjacent interval distance ratio between heads and slots is not always an integer, the approximate value also improves productivity by shortening the pickup distance of the gantry. Finally, since the algorithm implementation is essentially a simulation of the picking process, it can be fine-tuned to offer a tailored solution, including but not limited to preassign feeders, assigning nozzle to head, and prohibiting feeder slots.

IV. Experimental Result Analysis

The algorithms proposed in this article are implemented in Python 3.8 by a desktop computer with Intel Core i7 1.8-GHz CPU and compared with aggregation mixed-integer programming (AMIP) [6], hybrid genetic algorithm (HGA) [9], cell division genetic algorithm (CDGA) [18], and optimizer integrated with an industrial software (ISO). Both HGA and CDGA are representatives of evolutionary algorithms for assembly optimization. AMIP, a mathematical programming technique combined with an aggregation technique, could optimize medium-sized

Algorithm 4: PAP Sequence Heuristic.

Input: PCB data with coordinate (X_p, Y_p) of point p, component assignment $\mathcal{C}$ and $\mathcal{K}$
Output: PAP sequence $\mathcal{P}$
Initialize $B = \{1, 2, \ldots, \beta\}$ as beam set where β is the beam width;
Initialize $\mathcal{P}, \mathcal{P}_b$ as empty matrix and $\mathcal{T}_b$ as $1 \times |H|$ matrix, $\forall b \in B$;
for $\mathcal{H}^{\mathrm{CP}} \in \mathcal{C}, k \in \mathcal{K}$ **do**
  while $k \neq 0$ **do**
    Initialize $\beta \times 2$ matrix $\mathcal{W}$ as the coordinates of the β leftmost unplaced points;
    for $h \in H$ **do**
      Select β points which nearest to $\mathcal{W}(b), \forall b \in B$ with component type $\mathcal{H}^{\mathrm{CP}}(h)$;
      Select β points among β^2 candidates with minimal Chebyshev distance as $p_1, \cdots, p_b$;
    end
    $k \leftarrow k - 1, \mathcal{W}_b \leftarrow [X_{p_b}, Y_{p_b} - (h-1) \cdot \rho]$, $\mathcal{T}_b(h) \leftarrow p_b, \forall b \in B$;
    PAP sequence schedule for $\mathcal{T}_b$ using dynamic programming and attach $\mathcal{T}_b$ to $\mathcal{P}_b$ with column direction, $\forall b \in B$;
  end
end
$\mathcal{P} \leftarrow \mathcal{P}_b$ with minimal Chebyshev distance $\forall b \in B$;

TABLE II
Comparison of the Proposed Algorithms and the MIP Model

PCB	Scale (N, C, P)	Objective value T_{scan}	T_{mip}	Gap (%)	Computation time t_{scan}	t_{mip}
1-1	$(1, 1, 14)$	4.735	4.408	7.42	0.29	323.60
1-2	$(2, 1, 14)$	4.314	3.833	12.55	0.34	34.03
1-3	$(3, 2, 16)$	4.095	3.886	5.83	0.20	984.10
1-4	$(4, 2, 20)$	4.720	4.165	13.33	0.27	1117.84
1-5	$(5, 3, 2)$	5.793	5.170	12.05	0.48	718.44
1-6	$(6, 3, 26)$	6.257	5.773	8.38	0.59	5445.63
AVG				9.93		

data in an acceptable amount of time. All the mathematical models mentioned in this article are solved using the optimizer Gurobi [24].

First, we compare the proposed algorithm to the optimal solution of the mixed-integer model, as shown in Table II. Based on the production result, the coefficients t_c, t_n, t_p, and t_m of the MIP model are set to 2, 6, 1, and 0.1, respectively. As the size of the problem increases, the model becomes less capable of solving the small-scale data in Table II. However, the solving efficiency of the proposed heuristic algorithms is substantially better than mathematical planning methods with an optimality gap of 9.93% average.

Second, we use several industrial PCB data, including a randomly generated complex one as representatives, to compare the result of different methods. The latter can be equated to a multibatch PCB assembly scenario without feeder setup change.

TABLE III
PCB Data Parameters

PCB	2-1	2-2	2-3	2-4	2-5	2-6	2-7	2-8
N	1	1	2	3	3	3	3	4
C	7	18	6	7	16	20	24	41
P	564	176	72	192	114	150	236	1510

TABLE IV
Subobjective Comparison

PCB	ISO	AMIP	HGA	CDGA	OUR
2-1	94,0,444[1]	95,0,490	420,0,444	94,0,432	95,0,490
2-2	30,0,56	30,0,115	36,0,54	40,0,86	30,0,52
2-3	16,0,22	16,0,48	16,0,16	16,0,24	16,0,22
2-4	32,1,74	38,0,122	64,0,80	48,0,80	32,1,64
2-5	20,0,37	20,0,78	24,0,30	24,0,30	20,0,30
2-6	26,2,98	32,2,94	33,0,108	81,3,84	32,0,96
2-7	42,1,68	–	46,0,62	44,4,102	45,0,64
2-8	290,9,552	–	370,0,425	280,9,812	288,2,440

[1] The comma-separated values represent the subobjectives of the number of cycles, nozzle changes, and pickups, respectively.

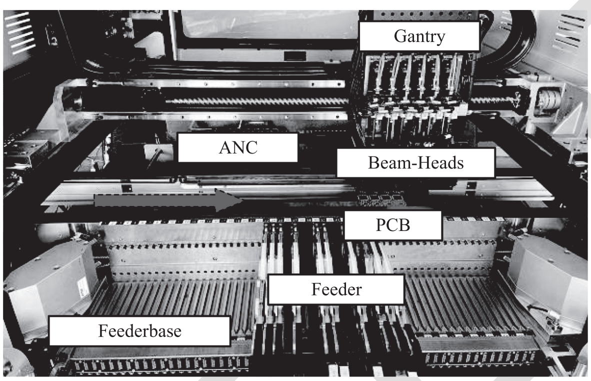

Fig. 4. Experimental platform of the placement machine.

TABLE V
CPH for Different Methods

	OUR	ISO		AMIP		HGA		CDGA	
PCB	E^1	E^2	ΔE^2	E^3	ΔE^3	E^4	ΔE^4	E^5	ΔE^5
2-1	11 297	11 255	0.37	6991	61.60	7035	60.35	10 673	5.84
2-2	16 058	16 003	0.35	11 460	40.21	14 958	7.36	12 462	28.86
2-3	12 451	11 998	3.78	9231	34.88	12 191	2.13	11 759	5.88
2-4	13 658	12 869	6.13	11 404	19.76	9795	39.43	10 423	31.03
2-5	13 375	13 022	2.72	9932	34.67	12 346	8.34	11 372	17.62
2-6	12 903	11 627	10.97	8843	45.91	10 457	23.39	7556	70.76
2-7	13 043	12 087	7.92	–	–	12 087	7.92	9830	32.69
2-8	13 557	11 835	14.55	–	–	11 781	15.08	10 477	29.40
AVG			5.85		39.49		20.50		27.76

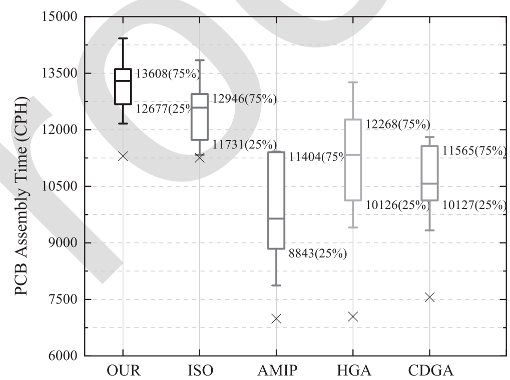

Fig. 5. Mounting time (CPH) distribution.

TABLE VI
Time Consuming of Different Methods

PCB	AMIP	HGA	CDGA	OUR
2-1	1.54	646.96	221.27	3.93
2-2	0.83	159.27	23.61	2.31
2-3	0.66	29.93	4.37	0.73
2-4	1.26	136.48	6.30	1.05
2-5	2.83	82.18	13.97	1.17
2-6	13.92	129.21	20.74	3.47
2-7	–	215.43	40.06	5.20
2-8	–	635.00	171.89	23.25
AVG	–	94.21	204.65	11.93

The comparative PCB data parameters are shown in Table III. According to the machine parameters, we set $e = 0.5$, $e_1 = 4$, and $e_2 = 0.6$ in the implementation of the heuristic algorithms. We set the size of the beam in the beam search to half the number of placement heads. This research investigates the effects of the optimization technique without feeder prearrangement since AMIP, HGA, and CDGA cannot deal with prearrangement conditions, and AMIP and HGA can only optimize single feeder type. The experiment findings indicate the suggested approach, ISO, AMIP, HGA, and CDGA, respectively, as $E^i (i = 1, 2, 3, 4, 5)$. The performance improvement of the suggested approach over other methods is represented by ΔE^i, which is computed as $\Delta E^i = (E^1 - E^i)/E^1 \times 100\%, i = 2, 3, 4, 5$.

This article compares the main subobjective values of optimization method results with each other, as shown in Table IV. The number of PAP cycles is one of the overall performance subobjectives since, in some cases, it may affect the distance of the moving route. The method proposed in this article exhibits more effective search capabilities when dealing with complex data.

Algorithm verification is done on our placement machine platform, which is shown in Fig. 4. We convert the assembly time into the standard time chip per hour (CPH) to provide a straightforward comparison independent of the number of placement points. A batch of PCBs is subjected to each procedure three times, and Table V shows the average assembly time. Even though the proposed algorithm does not significantly outperform the industrial customize optimizer results for small- and medium-sized data, its advantages become more evident as the size of the problem increases. The assembly efficiency distribution shown in Fig. 5 shows that the proposed algorithm is more stable than others.

The search efficiency is compared with other methods in Table V except for the built-in industrial customize optimizer. It can be seen that evolutionary-based algorithms take a longer time to find a solution, and the results are usually unstable due to their random exploration. AMIP is still intractable for large-scale PCB data, despite the efficient aggregate-based technique incorporated.

TABLE VII
CPH FOR DIFFERENT METHODS WITH MULTIWIDTH FEEDERS

	Parameter			Objective value		
PCB	P	C	N	E^1	E^2	ΔE^2
3-1	78	16	3	10 912	10 688	2.06
3-2	150	20	3	8493	8229	3.11
3-3	110	23	3	13 001	12 811	1.46
3-4	161	38	3	11 143	8798	21.04
3-5	540	10	4	8416	7548	10.31
AVG						7.60

The feeder allocation has a pivotal impact on the overall assembly efficiency, but only some researchers elaborate on the solution to the feeder types with different widths. We conduct comparative tests with PCB data using different width feeders to compare the suggested approach with the ISO method. According to Table VII, the proposed method provides a 7.60% overall efficiency gain over the industrial customize optimizer.

V. CONCLUSION

The scan-based hierarchical heuristic algorithm demonstrated excellent performance and efficient search in solving the complex surface mount optimization problem. We proposed a mixed integer mathematical model and elaborately designed heuristic algorithms. The component pickup procedure inspired the techniques of feeder allocation and component assignment with linear-aligned heads. While the component assignment heuristic algorithm concentrated on multihead pickup, the heuristic feeder allocation approach emphasized feeder allocation, increasing simultaneous pickup numbers. The ultimate goals of both the algorithms were to improve pickup efficiency and decrease nozzle change. In this article, beam search was used to improve the search quality of the PAP route schedule. In terms of extension, the algorithm analyzed the requirements in various application scenarios and gave supporting solutions to be indeed applied to industrial production environments. The experiments compared several previous research and an industrial optimizer, and the findings demonstrated that the suggested technique considerably increased the efficiency of placement machine assembly.

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